Modelling Software-based Systems

Lecture 2 Proof Obligation Generation

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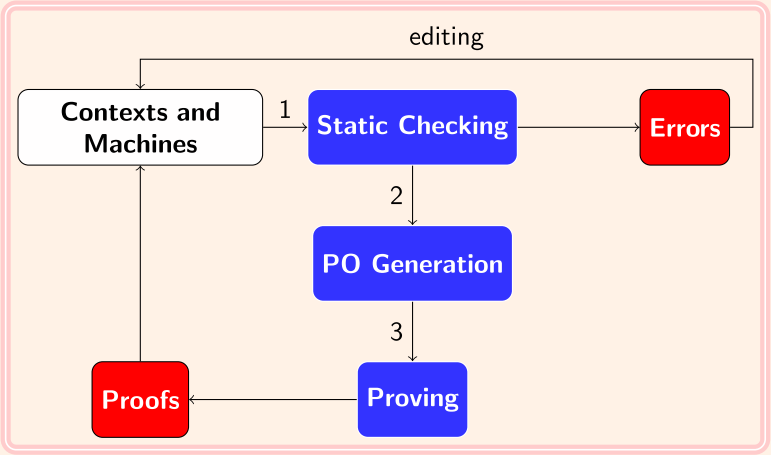
General Summary

1. Overview of machines, contexts and proof obligations
2. Proof Obligations for Contexts and Machines
   - PO thm/THM (context)
   - PO th/THM (machine)
   - PO evt/inv/INV
   - PO evt/act/FIS
3. Proof Obligations for Refinement
   - PO evt/grd/GRD
   - PO evt/act/SIM
   - PO evt/NAT
   - PO NAT
   - PO evt/VAR (arithmetic)
   - PO evt/VAR (set-theoretic)
   - PO evt/x/WFIS

Current Summary

1. Overview of machines, contexts and proof obligations
2. Proof Obligations for Contexts and Machines
3. Proof Obligations for Refinement

Analysis of the Event-B Models



Machines en Event B

MACHINE
m
REFINES
am
SEES
c
VARIABLES
u
INVARIANTS
$I(s, c, u)$
THEOREMS
$Q(s, c, u)$
VARIANT
$exp(s, c, u)$
EVENTS
$INITIALIZATION$
$\ldots$
e
$\ldots$
END

Machines en Event B

MACHINE
m
REFINES
am
SEES
c
VARIABLES
u
INVARIANTS
$I(s,c,u)$
THEOREMS
$Q(s,c,u)$
VARIANT
$exp(s,c,u)$
EVENTS
$INITIALIZATION$
$\ldots$
e
$\ldots$
END

- $\Gamma(m)$: environment for the machine m defined by the context c and it provides a list of seen axioms $Ax(s,c)$ and a list of seen theorems $Th(s,c)$ for the sets s and constants c.
- $\Gamma(m) \vdash \forall u.\textsc{Init}(s,c,u) \Rightarrow \mathrm{I}(s,c,u)$
- For each event e in E : $\Gamma(m) \vdash \forall u,u'.\mathrm{I}(s,c,u) \wedge BA(e)(u,u') \Rightarrow \mathrm{I}(u')$
- For each event e in E : $\Gamma(m) \vdash \forall u.\mathrm{I}(s,c,u) \wedge GRD(e)(s,c,u) \Rightarrow \exists u'.BA(e)(u,u')$
- $\Gamma(m) \vdash \forall u.\mathrm{I}(s,c,u) \Rightarrow \mathrm{Q}(s,c,u)$
- Generated proof obligations are derived from those conditions.

Checking the well formation of Event-B expressions

- Event-B expressions are contexts, machines, properties, equations, set-theoretical expressions ...
- e is an Event-B expression and $\mathsf{wd}(e)$ is a logical property expressing the well definition of e.
- $\mathsf{wd}(1=2) \triangleq \mathsf{wd}(1) \wedge \mathsf{wd}(2)$
- $\mathsf{wd}(a/b) \triangleq b \neq 0 \wedge \mathsf{wd}(a) \wedge \mathsf{wd}(b)$
- $\mathsf{wd}(f(g)) \triangleq g \in dom(f) \wedge f \in A \nrightarrow B$

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PO th/THM

CONTEXTS
  c
EXTENDS
  ac
SETS
  s
CONSTANTS
  c
AXIOMS
  $Ax(s,c)$
THEOREMS
  $th_1 : P_1(s,c)$
  $\ldots$
  $th_n : P_n(s,c)$
  $th : P(s,c)$
  $\ldots$
END

s	*seen sets*
c	*seen constants*
$Ax(s,c)$	*seen axioms*
$Th(s,c)$	*previous proved theorems*
	$PTh(s,c) = \{P_i(s,c) \mid i \ 1..n\}$
$P(s,c)$	*property over s and c*

PO th/THM

$$Ax(s,c), Th(s,c) \vdash P(s,c)$$

PO th/THM (machine)

MACHINE
m
...
VARIABLES
u
INVARIANTS
$I(s, c, u)$
THEOREMS
$Q(s, c, u)$
$th : P(s, c, u)$
...
END

s	*seen sets*
c	*seen constants*
u	*variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*invariants*
$Q(s, c, u)$	*theorems*
$P(s, c, u)$	*property over s,c and u*

PO th/THM

$$Ax(s, c), Th(s, c), I(s, c, u) \vdash P(s, c, u)$$

PO evt/inv/INV

```
EVENT evt
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :| BAP(x, s, c, u, u')
  END
```

$BA(\text{evt}) \;\widehat{=}$
$$\exists x. \begin{pmatrix} \wedge\, G(x,s,c,u) \\ \wedge\, BAP(x,s,c,u,u') \end{pmatrix}$$
$GRD(\text{evt}) \;\widehat{=}\; G(x,s,c,u)$
$ACT(\text{evt}) \;\widehat{=}\; BAP(x,s,c,u,u')$

s	*seen sets*
c	*seen constants*
u	*variables*
$Ax(s,c)$	*seen axioms*
$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*invariants*
$Q(s,c,u)$	*theorems*
evt	*event name*
x	*event parameter*
$G(x,s,c,u)$	*event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*
$inv : inv(s,c,u')$	*specific modified invariant*

PO evt/inv/INV

$$Ax(s,c), Th(s,c), I(s,c,u), G(x,s,c,u), BAP(x,s,c,u,u') \vdash inv(s,c,u')$$

PO Q/THM $Ax(s,c), Th(s,c), I(s,c,u) \vdash Q(s,c,u)$

PO evt/act/FIS

```
EVENT evt
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :| BAP(x, s, c, u, u')
  END
```

$BA(\mathsf{evt}) \ \widehat{=}$

$$\left(\begin{array}{l} \wedge\ G(x,s,c,u) \\ \wedge\ BAP(x,s,c,u,u') \end{array} \right)$$

$GRD(\mathsf{evt}) \ \widehat{=}\ G(x,s,c,u)$

$ACT(\mathsf{evt}) \ \widehat{=}$

$BAP(x,s,c,u,u')$

s	*seen sets*
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x	*event parameter*
$G(x,s,c,u)$	*event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*

PO evt/act/FIS

$$Ax(s,c), Th(s,c), I(s,c,u), G(x,s,c,u), \vdash \exists u'.BAP(x,s,c,u,u')$$

Current Summary

1. Overview of machines, contexts and proof obligations
2. Proof Obligations for Contexts and Machines
3. Proof Obligations for Refinement

PO evt/grd/GRD

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :| ABAP(x, s, c, u, u')
  END
```

```
EVENT ce
  REFINES
    ae
  ANY y WHERE
    H(y, s, c, v)
  WITH
    x : W(x, y, s, c, v)
  THEN
    v :| CBAP(y, s, c, v, v')
  END
```

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u),\ R(s, c, u, v)$	*abstract and concrete theorems*
ae, ce	*abstract and concrete event name*
x,y	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$H(y, s, c, v)$	*concrete event guard*
$ABAP(x, s, c, u, u')$	*abstract event before-after predic*
$CBAP(x, s, c, u, u')$	*concrete event before-after predic*
W(x,y,s,c,v)	witness predicate

PO evt/grd/GRD

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), W(x, y, s, c, v), H(y, s, c, v), \vdash G(x, s, c, u')$

PO evt/act/SIM

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :| ABAP(x, s, c, u, u')
  END
```

```
EVENT ce
  REFINES
    ae
  ANY y WHERE
    H(y, s, c, v)
  WITH
    x : WP(x, y, s, c, v)
    u' : WV(y, u', s, c, v)
  THEN
    v :| CBAP(y, s, c, v, v')
  END
```

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$ABAP(x,s,c,u,u')$	*abstract event before-after predic*
$CBAP(x,s,c,u,u')$	*concrete event before-after predic*
$WP(x,y,s,c,v)$	witness parameter predicate
$WV(y,u',s,c,v)$	witness variable predicate

PO evt/act/SIM

$$\left(\begin{array}{l} Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v) \\ WP(x,y,s,c,v), WV(y,u',s,c,v) \\ H(y,s,c,v), CBAP(y,s,c,v,v') \end{array}\right) \vdash ABAP(x,s,c,u,u')$$

PO evt/act/SIM

EVENT ae
  ANY x **WHERE**
    $G(x,s,c,u)$
  THEN
    $u :| BAP(x,s,c,u,u')$
  END
...
VARIANT
    $exp(s,c,u)$

s	*seen sets*
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$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*abstract invariants*
$J(s,c,u,v)$	*concrete invariants*
$Q(s,c,u),\ R(s,c,u,v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x,s,c,u)$	*abstract event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*
$exp(s,c,u)$	*aritthmetic expression*

PO evt/NAT

$$Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), G(x,s,c,u) \vdash exp(s,c,u) \in \mathbb{N}$$

PO evt/act/SIM

EVENT ae
  ANY x **WHERE**
    $G(x,s,c,u)$
  THEN
    $u :| BAP(x,s,c,u,u')$
  END
...
VARIANT
    $exp(s,c,u)$

s	*seen sets*
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$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*abstract invariants*
$J(s,c,u,v)$	*concrete invariants*
$Q(s,c,u),\ R(s,c,u,v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x,s,c,u)$	*abstract event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*
$setexp(s,c,u)$	*set expression*

PO evt/NAT $Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), G(x,s,c,u) \vdash$
$finite(setexp(s,c,u))$

PO evt/VAR

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :|BAP(x, s, c, u, u')
  END
...
VARIANT
    exp(s, c, u)
```

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u),\ R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$exp(s, c, u)$	*aritthmetic expression*

PO evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash exp(s, c, u') < exp(s, c, u)$$

PO evt/VAR

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :| BAP(x, s, c, u, u')
  END
...
VARIANT
    setexp(s, c, u)
```

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$Th(s, c)$	*seen theorems*
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$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u),\ R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$setexp(s, c, u)$	*set-theoretic expression*

PO evt/VAR

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash$
$setexp(s, c, u') \subset setexp(s, c, u)$

PO evt/x/WFIS

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :| ABAP(x, s, c, u, u')
  END
```

```
EVENT ce
  REFINES
    ae
  ANY y WHERE
    H(y, s, c, v)
  WITH
    x : WP(x, y, s, c, v)
    u' : WV(y, u', s, c, v)
  THEN
    v :| CBAP(y, s, c, v, v')
  END
```

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$CBAP(x, s, c, u, u')$	*concrete event before-after predic*
$WP(x, y, s, c, v)$	witness parameter predicate
$WV(y, u', s, c, v)$	witness variable predicate

PO evt/x/WFIS

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), H(y, s, c, v) \vdash$
$\exists x . WP(x, y, s, c, v)$