Modelling Software-based Systems

Lecture 2

Proof Obligation Generation

Master Informatique

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General Summary

❶ Overview of machines, contexts and proof obligations

❷ Proof Obligations for Contexts and Machines

- PO thm/THM (context)
- PO thm/THM (machine)
- PO evt/inv/INV
- PO evt/act/FIS

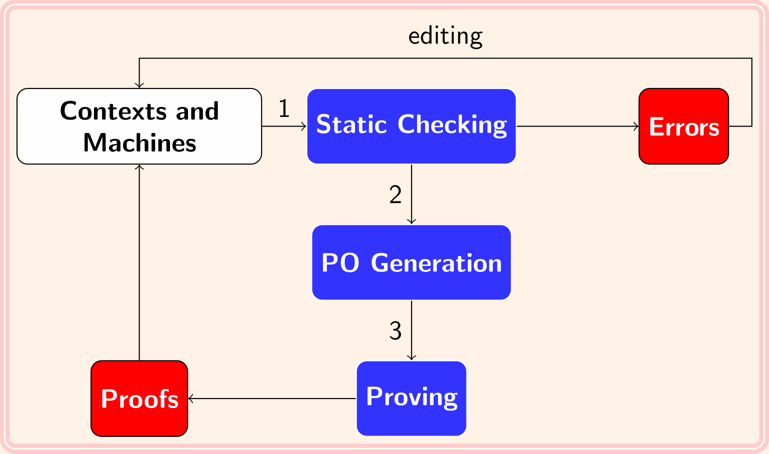
❸ Proof Obligations for Refinement

- PO evt/grd/GRD
- PO evt/act/SIM
- PO evt/NAT
- PO NAT
- PO evt/VAR (arithmetic)
- PO evt/VAR (set-theoretic)
- PO evt/x/WFIS

Current Summary

1. Overview of machines, contexts and proof obligations
2. Proof Obligations for Contexts and Machines
3. Proof Obligations for Refinement

Analysis of the Event-B Models



Machines en Event B

MACHINE
  m
REFINES
  am
SEES
  c
VARIABLES
  u
INVARIANTS
  $I(s,c,u)$
THEOREMS
  $Q(s,c,u)$
VARIANT
  $exp(s,c,u)$
EVENTS
  $INITIALIZATION$
  $\ldots$
  e
  $\ldots$
END

Machines en Event B

MACHINE

m

REFINES

am

SEES

c

VARIABLES

u

INVARIANTS

$I(s,c,u)$

THEOREMS

$Q(s,c,u)$

VARIANT

$exp(s,c,u)$

EVENTS

$INITIALIZATION$

$\ldots$

e

$\ldots$

END

- $\Gamma(m)$: environment for the machine m defined by the context c and it provides a list of seen axioms $Ax(s,c)$ and a list of seen theorems $Th(s,c)$ for the sets s and constants c.
- $\Gamma(m) \vdash \forall u.\textsc{Init}(s,c,u) \Rightarrow \mathrm{I}(s,c,u)$
- For each event e in E : $\Gamma(m) \vdash \forall u,u'.\mathrm{I}(s,c,u) \wedge BA(e)(u,u') \Rightarrow \mathrm{I}(u')$
- For each event e in E : $\Gamma(m) \vdash \forall u.\mathrm{I}(s,c,u) \wedge GRD(e)(s,c,u) \Rightarrow \exists u'.BA(e)(u,u')$
- $\Gamma(m) \vdash \forall u.\mathrm{I}(s,c,u) \Rightarrow \mathrm{Q}(s,c,u)$
- Generated proof obligations are derived from those conditions.

Current Summary

1. Overview of machines, contexts and proof obligations
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PO thm/THM

CONTEXTS
  c
EXTENDS
  ac
SETS
  s
CONSTANTS
  c
AXIOMS
  $Ax(s,c)$
THEOREMS
  $th_1 : P_1(s,c)$
  $\ldots$
  $th_n : P_n(s,c)$
  $th : P(s,c)$
  $\ldots$
END

s	*seen sets*
c	*seen constants*
$Ax(s,c)$	*seen axioms*
$Th(s,c)$	*previous proved theorems*
	$Th(s,c) = \{P_i(s,c) \vert i\ 1..n\}$
$P(s,c)$	*property over s and c*

PO th/THM

$$Ax(s,c), Th(s,c) \vdash P(s,c)$$

PO thm/THM (machine)

MACHINE

m

$\dots$

VARIABLES

u

INVARIANTS

$I(s,c,u)$

THEOREMS

$Q(s,c,u)$

$th : P(s,c,u)$

$\dots$

END

s	*seen sets*
c	*seen constants*
u	*variables*
$Ax(s,c)$	*seen axioms*
$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*invariants*
$Q(s,c,u)$	*theorems*
$P(s,c,u)$	*property over s,c and u*

PO thm/THM

$$Ax(s,c), Th(s,c), I(s,c,u) \vdash P(s,c,u)$$

PO evt/inv/INV

```
EVENT evt
  ANY $x$ WHERE
    $G(x,s,c,u)$
  THEN
    $u :| BAP(x,s,c,u,u')$
  END
```

$BA(\mathsf{evt}) \ \widehat{=}$

$\exists x. \left(\begin{array}{l} \wedge\ G(x,s,c,u) \\ \wedge\ BAP(x,s,c,u,u') \end{array} \right)$

$GRD(\mathsf{evt}) \ \widehat{=} \ G(x,s,c,u)$

$ACT(\mathsf{evt}) \ \widehat{=} \ BAP(x,s,c,u,u')$

s	*seen sets*
c	*seen constants*
u	*variables*
$Ax(s,c)$	*seen axioms*
$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*invariants*
$Q(s,c,u)$	*theorems*
evt	*event name*
x	*event parameter*
$G(x,s,c,u)$	*event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*
$inv : inv(s,c,u')$	*specific modified invariant*

PO evt/inv/INV

$$Ax(s,c), Th(s,c), I(s,c,u), G(x,s,c,u), BAP(x,s,c,u,u') \vdash inv(s,c,u')$$

PO Q/THM $Ax(s,c), Th(s,c), I(s,c,u) \vdash Q(s,c,u)$

PO evt/act/FIS

```
EVENT evt
  ANY $x$ WHERE
    $G(x,s,c,u)$
  THEN
    $u :| BAP(x,s,c,u,u')$
  END
```

$$BA(\mathsf{evt}) \ \widehat{=} \left(\begin{array}{l} \wedge\ G(x,s,c,u) \\ \wedge\ BAP(x,s,c,u,u') \end{array} \right)$$

$$GRD(\mathsf{evt}) \ \widehat{=} \ G(x,s,c,u)$$

$$ACT(\mathsf{evt}) \ \widehat{=}$$
$$BAP(x,s,c,u,u')$$

s	*seen sets*
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u	*variables*
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$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*invariants*
$Q(s,c,u)$	*theorems*
evt	*event name*
x	*event parameter*
$G(x,s,c,u)$	*event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*

PO evt/act/FIS

$$Ax(s,c), Th(s,c), I(s,c,u), G(x,s,c,u), \vdash \exists u'.BAP(x,s,c,u,u')$$

Current Summary

1. Overview of machines, contexts and proof obligations

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3. Proof Obligations for Refinement

PO evt/grd/GRD

```
EVENT ae
  ANY $x$ WHERE
    $G(x,s,c,u)$
  THEN
    $u :| ABAP(x,s,c,u,u')$
  END
```

```
EVENT ce
  REFINES
     $ae$
  ANY $y$ WHERE
    $H(y,s,c,v)$
  WITH
    $x : W(x,y,s,c,v)$
  THEN
    $v :| CBAP(y,s,c,v,v')$
  END
```

s	*seen sets*
c	*seen constants*
u,v	*abstract and concrete variables*
$Ax(s,c)$	*seen axioms*
$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*abstract invariants*
$J(s,c,u,v)$	*concrete invariants*
$Q(s,c,u),\ R(s,c,u,v)$	*abstract and concrete theorems*
ae, ce	*abstract and concrete event name*
x,y	*event parameters*
$G(x,s,c,u)$	*abstract event guard*
$H(y,s,c,v)$	*concrete event guard*
$ABAP(x,s,c,u,u')$	*abstract event before-after predic*
$CBAP(x,s,c,u,u')$	*concrete event before-after predic*
W(x,y,s,c,v)	witness predicate

PO evt/grd/GRD

$$Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), W(x,y,s,c,v), H(y,s,c,v), \vdash G(x,s,c,u')$$

PO evt/act/SIM

```
EVENT ae
  ANY $x$ WHERE
    $G(x,s,c,u)$
  THEN
    $u :| ABAP(x,s,c,u,u')$
  END
```

```
EVENT ce
  REFINES
    $ae$
  ANY $y$ WHERE
    $H(y,s,c,v)$
  WITH
    $x : WP(x,y,s,c,v)$
    $u' : WV(y,u',s,c,v)$
  THEN
    $v :| CBAP(y,s,c,v,v')$
  END
```

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$ABAP(x,s,c,u,u')$	*abstract event before-after predic*
$CBAP(x,s,c,u,u')$	*concrete event before-after predic*
$WP(x,y,s,c,v)$	witness parameter predicate
$WV(y,u',s,c,v)$	witness variable predicate

PO evt/act/SIM

$$\left(\begin{array}{l} Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v) \\ WP(x,y,s,c,v), WV(y,u',s,c,v) \\ H(y,s,c,v), CBAP(y,s,c,v,v') \end{array}\right) \vdash ABAP(x,s,c,u,u')$$

PO evt/act/SIM

```
EVENT ae
  ANY $x$ WHERE
    $G(x,s,c,u)$
  THEN
    $u :| BAP(x,s,c,u,u')$
  END
...
VARIANT
     $exp(s,c,u)$
```

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$Th(s,c)$	*seen theorems*
$I(s,c,u)$	*abstract invariants*
$J(s,c,u,v)$	*concrete invariants*
$Q(s,c,u),\ R(s,c,u,v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x,s,c,u)$	*abstract event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*
$exp(s,c,u)$	*aritthmetic expression*

PO evt/NAT

$$Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), G(x,s,c,u) \vdash exp(s,c,u) \in \mathbb{N}$$

PO evt/act/SIM

```
EVENT ae
  ANY $x$ WHERE
    $G(x, s, c, u)$
  THEN
    $u :| BAP(x, s, c, u, u')$
  END
...
VARIANT
    $exp(s, c, u)$
```

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$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u),\ R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$setexp(s, c, u)$	*set expression*

PO evt/NAT $Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), G(x,s,c,u) \vdash$

$finite(setexp(s,c,u))$

PO evt/VAR

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :|BAP(x, s, c, u, u')
  END
...
VARIANT
    exp(s, c, u)
```

s	*seen sets*
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u, v	*abstract and concrete variables*
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$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u),\ R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$exp(s, c, u)$	*aritthmetic expression*

PO evt/VAR

$$Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), G(x,s,c,u), BAP(x,s,c,u,u') \vdash exp(s,c,u') < exp(s,c,u)$$

PO evt/VAR

```
EVENT ae
  ANY $x$ WHERE
    $G(x,s,c,u)$
  THEN
    $u :|BAP(x,s,c,u,u')$
  END
...
VARIANT
    $setexp(s,c,u)$
```

s	*seen sets*
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u, v	*abstract and concrete variables*
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$Th(s,c)$	*seen theorems*
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$J(s,c,u,v)$	*concrete invariants*
$Q(s,c,u),\ R(s,c,u,v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x,s,c,u)$	*abstract event guard*
$BAP(x,s,c,u,u')$	*event before-after predicate*
$setexp(s,c,u)$	*set-theoretic expression*

PO evt/VAR

$$Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), G(x,s,c,u), BAP(x,s,c,u,u') \vdash$$
$$setexp(s,c,u') \subset setexp(s,c,u)$$

PO evt/x/WFIS

```
EVENT ae
  ANY $x$ WHERE
    $G(x,s,c,u)$
  THEN
    $u :| ABAP(x,s,c,u,u')$
  END
```

```
EVENT ce
  REFINES
    $ae$
  ANY $y$ WHERE
    $H(y,s,c,v)$
  WITH
    $x : WP(x,y,s,c,v)$
    $u' : WV(y,u',s,c,v)$
  THEN
    $v :| CBAP(y,s,c,v,v')$
  END
```

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$CBAP(x,s,c,u,u')$	*concrete event before-after predic*
$WP(x,y,s,c,v)$	witness parameter predicate
$WV(y,u',s,c,v)$	witness variable predicate

PO evt/x/WFIS

$$Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v), H(y,s,c,v) \vdash$$
$$\exists x. WP(x,y,s,c,v)$$