Dependable Hybrid Systems Design: Prelude

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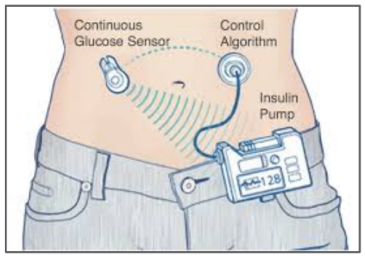
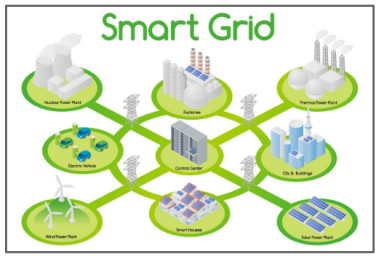
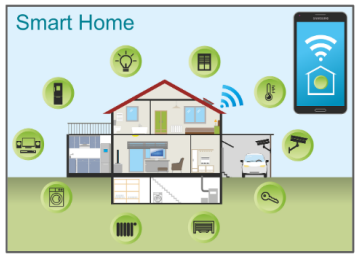
Outlines

Hybrid Systems

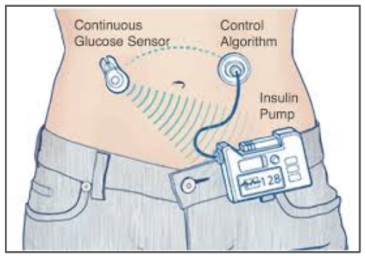


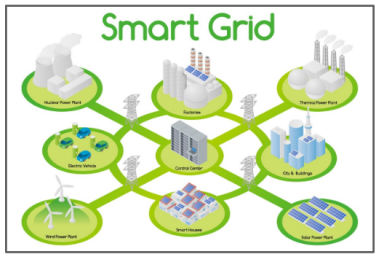


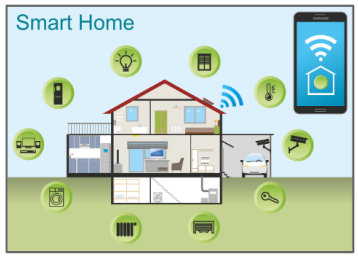


- Hybrid = Continuous + Discrete behaviors

Hybrid Systems

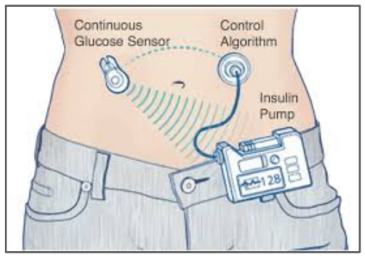


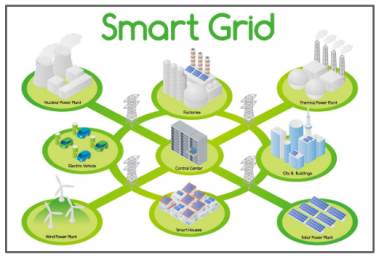


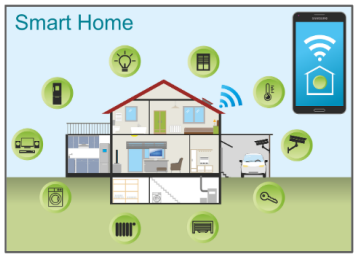


- Hybrid = Continuous + Discrete behaviors
- Open-loop vs. Close-loop

Hybrid Systems







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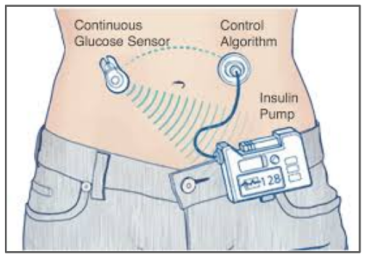
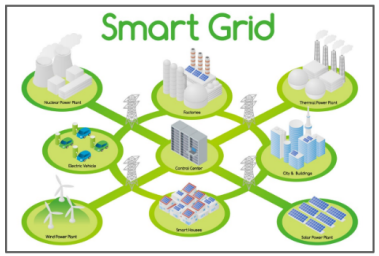
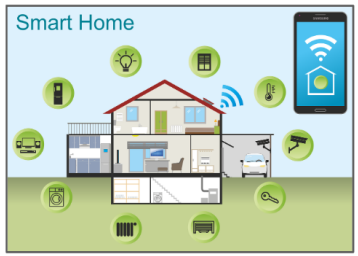
Hybrid Systems

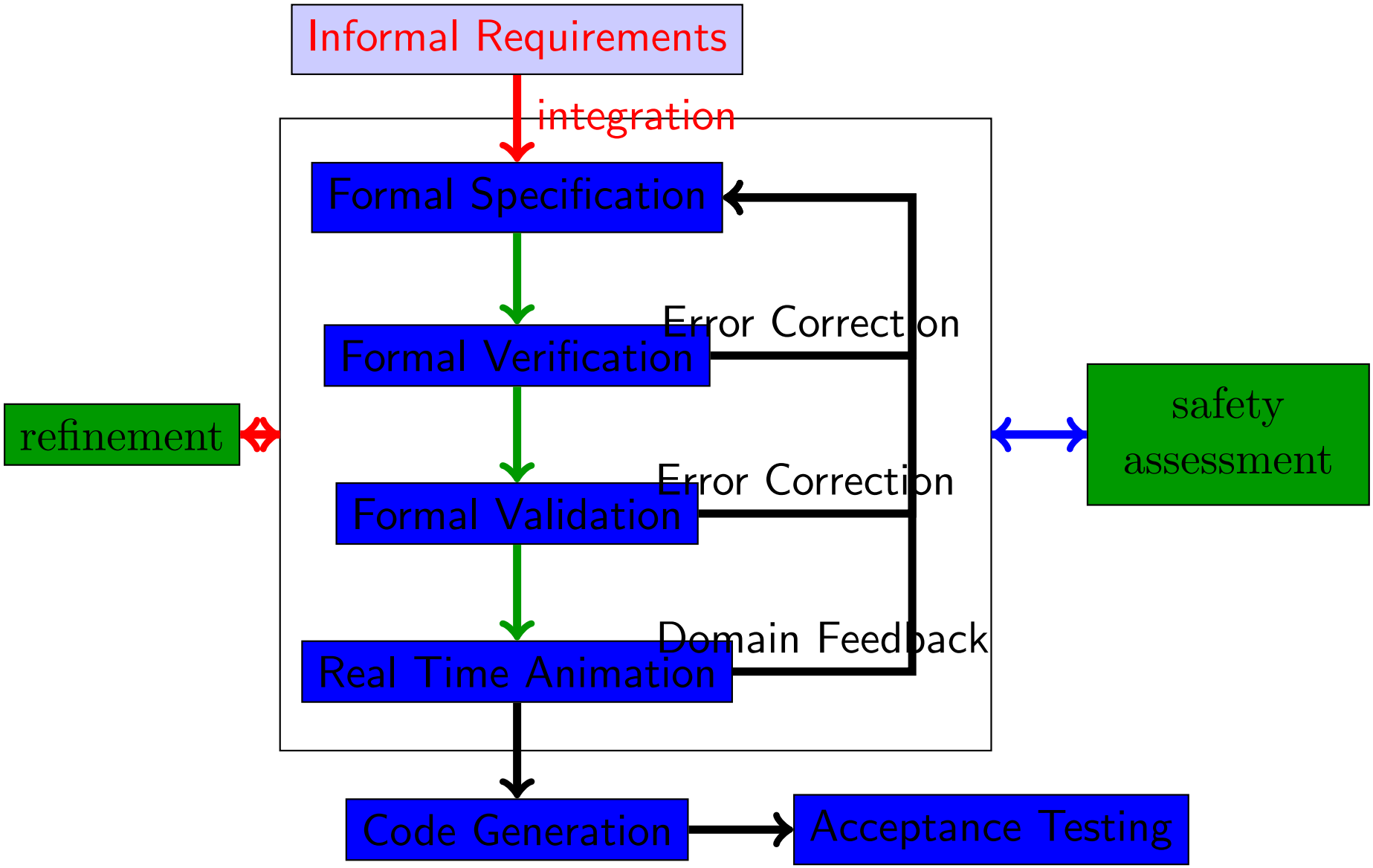






- Hybrid = Continuous + Discrete behaviors
- Open-loop vs. Close-loop
- Ubiquitous
- Safety

Tool Support for Design Safe Hybrid Systems



Tool Support for Design Safe Hybrid Systems

- Requirement engineering(formal specification)
- Theorem Proving(formal verification)
- Model Checking(formal validation)
- Simulation(real-time animation)

Requirement engineering

- Workflow
  - Rewriting informal requirement into formal specification(preconditions and postconditions)
- e.g. Event-B, Z, TLA+

Theroem proving

- Workflow
  - Hybrid system safety as a theorem:
    {Precondition} System {Postcondition}
  - Interacting with a mechanized theorem prover to generate proofs
- e.g. KeYmaera, HHL, Z3
- Expensive, time consuming

Model Checking

- Workflow
  - Building model, specify its initial states of interest, and safety property
  - Checking every execution of model starting in an initial state always stays within the set of safe states
- e.g. HyTech, SpaceEx, Flow*
- Undecidable in general

Simulation

- Workflow
  - Building model(behaviors, environment, interactions)
  - Computational evaluation of a model instance over time
  - Checking evaluation result
- e.g. Ptolemy, Matlab(Simulink,Stateflow), SAMSON
- Inconclusive result (valid only on the chosen input)

Outlines

Derivatives

- The rate of change of function f(x), w.r.t. x

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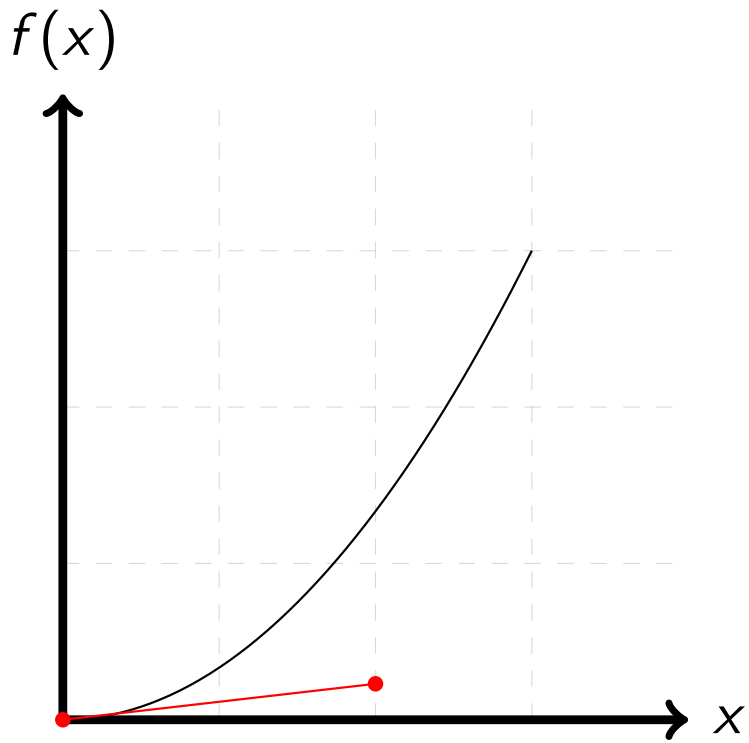
- Slope of the tangent line!



Derivatives

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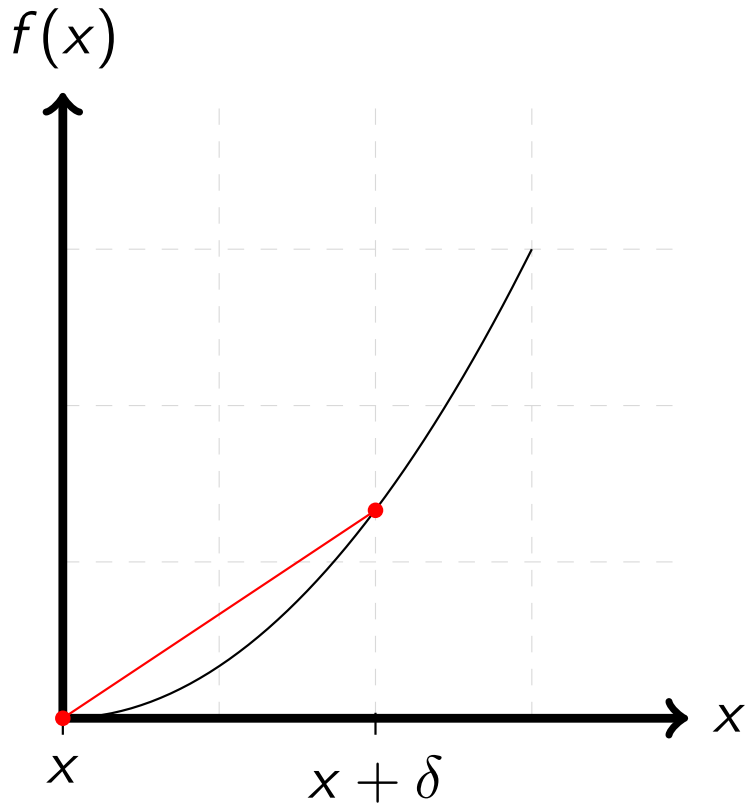
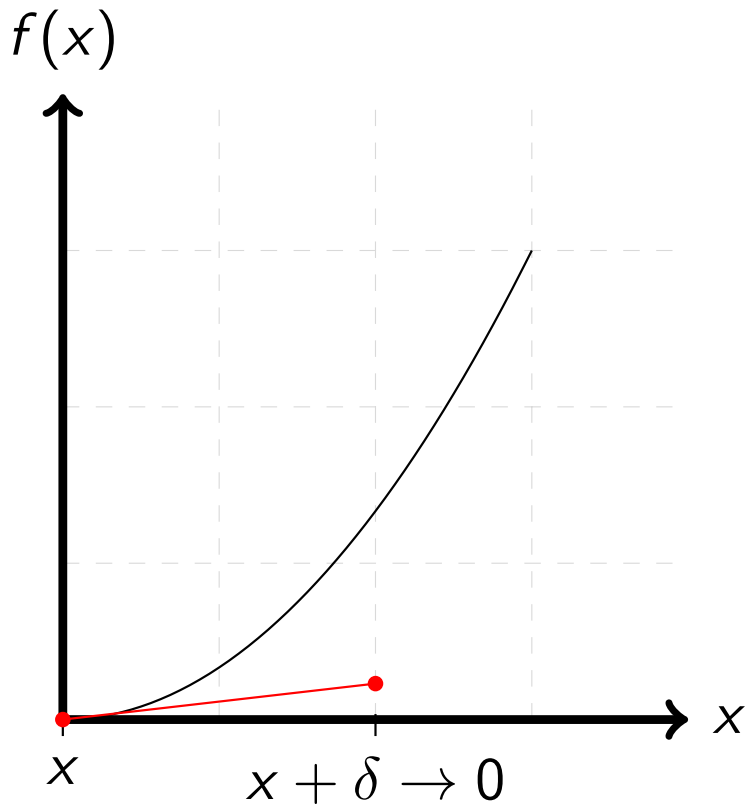
$$\frac{df}{dx} \approx \frac{f(x+\delta) - f(x)}{\delta}$$



Derivatives

- The rate of change of function f(x), w.r.t. x
- Slope of the tangent line!
-

$$\frac{df}{dx} = \lim_{\delta \to 0} \frac{f(x+\delta) - f(x)}{\delta}$$



Power Rule

Try $f(x) = x^n$, (Power Rule): $\frac{df}{dx} = nx^{n-1}$

$$\frac{df}{dx} \approx \frac{f(x+\delta) - f(x)}{\delta} \quad \textbf{(approximation)}$$

$$= \frac{1}{\delta}[(x+\delta)^n - x^n] \quad \textbf{(rewriting)}$$

$$= \frac{1}{\delta}(x^n + nx^{n-1}\delta + \frac{n(n-1)}{2}x^{n-2}\delta^2 \ldots - x^n) \quad \textbf{(Pascal-triangle)}$$

$$= \frac{1}{\delta}(nx^{n-1}\delta + O(\delta^2))$$

$$= nx^{n-1} + O(\delta)$$

Chain Rule

$\frac{d}{dx}(f(g(x))) = \dot{f}(g(x)) \cdot \dot{g}(x)$

Ex:

- f(x) = $\cos(x^3)$
- f(x) = 2^x
- f(x) = e^{x^2} sin(x)

Differential Equations

Ex: x is the size of a population of procreate bunnies...

- Population grows at a rate λ proportional to its population size: $\frac{dx}{dt} = \lambda x$
- What is population as a function over time?

$$\frac{dx}{dt} = \lambda x$$
$$\rightarrow \frac{dx}{x} = \lambda dt$$
$$\rightarrow \int \frac{dx}{x} = \int \lambda dt$$
$$\rightarrow ln(x) = \lambda t + C$$
$$\rightarrow x = e^{\lambda t + C}$$

- Determine C by initial conditions, e.g. (x_0 at t_0)

Outlines

Review of Event-B

- Context: static properties of Event-B models
  - Sets: user-defined types
  - Constants: static object in development
  - Axioms: presumed properties about sets and constants
  - Theorems: derived properties about sets and constants

Review of Event-B

- Machine: behavioral properties of Event-B models
  - Variables: states
  - Invariants: properties of variables that always need to hold
  - Theorems: derived properties about variables
  - Events: possible state changes

Review of Event-B

- Proof obligations: must be proved to show that Event-B models fulfill their specified properties.
  - INV: invariant preservation
  - FIS: action feasibility
  - ...

Develop Theories in Event-B

- Theory plugin: more modularize and reusable polymorphic "Context"
- Developed at University of Southampton, still under development
- Installation: `http://rodin-b-sharp.sourceforge.net/updates`
  - Modelling Extensions $\rightarrow$ Theory Feature
- Let us develop a theory for real numbers
- Fork: `https://github.com/veriatl/LORIA_WEEK1`
  - Open model "theory-axiom-reals"

Exercise One (*)

- Prove: $a + b + c = c + b + a$ on real numbers

? How to write this theorem

? What is the key to prove this theorem

? How to use theory plugin to prove this

Exercise Two (**)

- Develop the power operator a^b

? What are its arguments and results

? What is its semantics

Exercise Three (***)

- Open model “ex-pattern-const-DE”

? What this model does

? What is its invariant

? What operators are needed to express this invariant, and what are their semantics

? How to prove your invariant

Caveats

- Axioms inconsistency $\rightarrow$ Introduce when necessary, Prove when you can
- Big fat theories $\rightarrow$ Modular theories